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Your Roll No. 2023

Sr. No. of Question Paper	:	**4532**	**E**
Unique Paper Code	:	32221401	
Name of the Paper	:	Mathematical Physics III	
Name of the Course	:	**B.Sc. (H) Physics**	
Semester/Annual	:	IV	
Duration : 3 Hours			Maximum Marks : 75

Instructions

1. Write your Roll Number on the top immediately on the receipt of the question paper.
2. Attempt five questions in all.
3. Question number 1 is compulsory. Attempt two questions each from section A and B.
4. The Principal Branch of argument of complex number z in all the questions is taken to be $-\pi < \theta \leq \pi$
5. Use the following definition for the Fourier transform of $f(x)$:
$$\mathcal{F}(f(x)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ikx}dx$$
6. Use the following definition for the Fourier Sine transform of $f(x)$:
$$\mathcal{F}_s(f(x)) = \sqrt{\frac{2}{\pi}}\int_{0}^{\infty} f(x)sin(kx)dx$$
7. Use the following definition for the Fourier Cosine transform of $f(x)$:
$$\mathcal{F}_c(f(x)) = \sqrt{\frac{2}{\pi}}\int_{0}^{\infty} f(x)cos(kx)dx$$
8. The definition of convolution of two functions $f(x)$ and $g(x)$ for Fourier transform is:
$$(f * g)(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(y)g(x-y)dy$$
9. Some useful Fourier and Laplace Transforms are given at the end.

Q1. Attempt **any *five*** parts. All parts carry equal marks. (5 × 3 = 15)

a) Obtain the rectangular form of the ellipse $|z + 3| + |z - 3| = 10$.

b) Find the principal branch of i^i.

c) Without evaluating the integral, show that $\left|\int_\Gamma \frac{dz}{1+z}\right| \leq \frac{3\pi}{4}$; where Γ is the arc of circle $|z| = 3$ from $z = 3$ to $z = 3i$ lying in the first quadrant.

d) Given, Laplace Transform, $\mathcal{L}(J_0(t)) = \frac{1}{\sqrt{1+s^2}}$, find the Laplace Transform, $\mathcal{L}(e^{-at}J_0(bt))$.

e) Find Inverse Laplace Transform of $\frac{1}{(s^2 + a^2)(s^2 + b^2)}$.

f) Find the Fourier Transform of $cos(ax)$ in terms of Dirac-delta functions.

g) If $g(x) = f(bx + a)$ and F(k) is the Fourier transform of $f(x)$, determine the Fourier transform of $g(x)$.

h) Solve the integral $\int_0^5 (sin\, x)\, \delta[(x - 2)(x - 4)]\, dx$

Section – A

Q2.

a) Using de-Moivre's theorem prove that:

$$cos(4\theta) = 8cos^4(\theta) - 8cos^2(\theta) + 1$$

and hence show that

$$cos\frac{\pi}{8} = \left(\frac{2 + \sqrt{2}}{4}\right)^{1/2}$$ 6, 2

b) Prove:

$$coth^{-1}z = \frac{1}{2} ln\left(\frac{z+1}{z-1}\right)$$ 7

Q3.

a) Prove that the function $u(x, y) = 2x(1 - y)$ is harmonic. Find its conjugate function $v(x, y)$ such that $f(z) = u + iv$ is analytic. 7

b) Evaluate the following integrals using Cauchy's Integral Formulae where C: $|z| = 3$

i. $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{(z^2+1)^2}dz,\ t > 0$

ii. $\oint_C \frac{\sin^6(z)}{\left(z-\frac{\pi}{6}\right)^3}dz$ 4, 4

Q4.

a) Use Residue theorem to evaluate **any one** integral of the following: 9

i. $\int_0^\infty \frac{x^2}{(x^2+9)(x^2+4)^2}dx$

ii. $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta + c\sin\theta};\ a^2 > b^2 + c^2$

(b). Expand the function $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent series valid for

i. $1 < |z| < 2$

ii. $|z-1| > 1$ 3, 3

Section – B

Q5.

a) Find the Fourier transform of

$$f(x) = \begin{cases} (1-x^2), |x| < 1 \\ 0, |x| > 1 \end{cases}$$ 7

b) Verify the Convolution theorem (Fourier transform) for the functions

$$f(x) = g(x) = e^{-x^2}$$ 8

Q6.

a) If Laplace transform of f(t) is $\mathcal{L}(s)$ then prove that Laplace transform of $\frac{f(t)}{t}$ is $\int_s^\infty \mathcal{L}(u)du$. Use this result to evaluate Laplace transform of $\frac{\sin(t)}{t}$. 8

b) Taking the Laplace Transform of $f(t) = \int_0^\infty \frac{x\sin(tx)}{1+x^2}dx$, show that $f(t) = \frac{\pi}{2}e^{-t}$ for $t > 0$. 7

Q7.

a) A particle moves along a line so that its displacement x from a fixed point O at any time t is given by $x''(t) + 4x'(t) + 5x(t) = 80$. Initial conditions are $x(0) = x'(0) = 0$. Using Laplace Transform, find its displacement at any time $t > 0$. 8

b) Prove that

$$\mathcal{F}^{-1}\left(\frac{1}{k^4+5k^2+4}\right) = \frac{\sqrt{2\pi}}{12}\left(2\,e^{-|x|} - e^{-2|x|}\right)$$ 7

Some useful Laplace Transforms:

$$\mathcal{L}(sin(at)) = \frac{a}{s^2+a^2}, Re(s) > 0, s \neq \pm ia$$

$$\mathcal{L}(cos(at)) = \frac{s}{s^2+a^2}, Re(s) > 0, s \neq \pm ia$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a}, Re(s) > a$$

$$\mathcal{L}(t^a) = \frac{\Gamma(a+1)}{s^{a+1}}, Re(s) > a$$

Useful Fourier Transform:

$$\mathcal{F}(e^{-ax^2}) = \frac{1}{\sqrt{2a}} e^{-k^2/(4a)}, a > 0$$

Useful Inverse Fourier Transform:

$$\mathcal{F}^{-1}\left(\frac{1}{a^2+k^2}\right) = \frac{\sqrt{2\pi}}{2a} e^{-a|x|}$$

also, $\mathcal{F}^{-1}[a\,g(k) + b\,h(k)] = a\,\mathcal{F}^{-1}[g(k)] + b\,\mathcal{F}^{-1}[h(k)]$

(a and b are constants)

Useful Integral:

$$\int_{-\infty}^{\infty} e^{-ax^2+bx}\,dx = e^{b^2/(4a)}\sqrt{\frac{\pi}{a}}\,;\ a > 0,\ b \text{ can be purely imaginary also.}$$

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Your Roll No. 2023

Sr. No. of Question Paper	:	**4688** **E**
Unique Paper Code	:	32221402
Name of the Paper	:	Elements of Modern Physics
Name of the Course	:	**B.Sc. (Hons) Physics –CBCS Core**
Semester	:	IV
Duration : 3 Hours		Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all
3. **Question 1** is compulsory
4. **All questions** carry equal marks
5. **Symbols** have their usual meanings
6. Use of **non-programmable** calculators is allowed

Answer **any five** questions from the following.

1. (a) The irradiated power of a body at 333^0C is 1000 J/s. If the temperature of this body is raised to 666°C, find the radiated power of the body.

   (b) What is the physical significance of a wave function? What conditions must be satisfied by an acceptable wave function?

   (c) What voltage must be applied to an electron microscope to produce electrons of wavelength 0.20Å.

(d)The spectral line of wavelength 450 nm has a width of 10^{-4} nm. Find the average time that the atomic system remains in the corresponding energy state?

(e) How is the time dependent Schrodinger wave equation obtained from the time independent Schrodinger equation.

(f) What inferences can be drawn from the single and double slit/s experiment with electrons?

(g) Find the penetration depth of an electron having kinetic energy 10 keV when it strikes a potential step of height 15 keV.

(h) Determine the approximate density of a nucleus treating it as a uniform sphere. (Given: mass of a nucleon = 1.7×10^{-27} kg.) (3×5=15)

2. a) Draw the Energy vs wavelength curve of a blackbody for three different temperatures $T_1 < T_2 < T_3$. Show that the Wein's law and Rayleigh Jeans law of black body radiation are the special cases of Planck's law. (5)

(b) The threshold wavelength of potassium is 558 nm. What is the work function for potassium? What is the stopping potential when light of 400 nm is incident on potassium? (5)

(c) Calculate the energy in electron volt of a photon of wavelength 10 Å. What is the momentum of this photon? (5)

3. (a) Distinguish between phase velocity and group velocity and obtain an expression for both. Derive the relation between them. (5)

(b) Show that the Compton wavelength is independent of the nature of the scatterer and the original wavelength of the incident beam. (5)

(c) An electron has a deBroglie wavelength equal to that of a photon, show that the ratio of the kinetic energy of the electron to the energy of photon is

$$\frac{(m^2c^4 + h^2\nu^2)^{\frac{1}{2}} - mc^2}{h\nu} \qquad (5)$$

4. (a)Show mathematically the value of wavelength calculated from the Davisson Germer experiment matches the value of the wavelength calculated from the deBroglie's hypothesis. (5)

(b) Estimate the minimum energy of a proton existing inside the nucleus using Heisenberg's uncertainty principle. (Size of the nucleus= 1×10^{-15}m) (5)

(c)Explain why it is plausible to define probability current density in quantum mechanics by the following expression

$$J= (e\hbar/2m)\ (\psi^* \text{ grad } \psi - \psi \text{ grad } \psi$$

The symbols have the usual meaning (5)

5. (a) What is quantum mechanical tunneling? Obtain an expression for the transmission probability for a beam of particles each with mass m and energy E ($E<V_0$) incident on a rectangular potential barrier:

$V(x) = 0$ for $x<0$

$= Vo$ for $0<x<a$

$= 0$ for $x>a$ (10)

(b) Obtain and draw the first two normalized wave functions for a particle in a one dimensional potential box. (5)

6 (a) What are nuclear forces and their characteristics? Also draw the N-Z plot and explain the stability of the nucleus. (5)

(b) Calculate the binding energy of an alpha particle from the following data in MeV and Joules. (Given: mass of He atom= 4.00260 amu, mass of neutron = 1.008665amu, mass of proton=1.007276amu) (5)

(c) Calculate the total energy released if 1.2 kg of 235 U undergoes fission, taking the disintegration energy per event to be Q = 208 MeV. (5)

7. (a) Explain why electron positron pair creation necessarily requires the presence of a nucleus. (5)

(b) Calculate the time required for 20% of a sample of thorium to disintegrate. Assume the half life of thorium to be 1.4×10^{10} years. Calculate the mean life time of thorium nucleus. (5)

(c) Bring out the differences between atomic absorption, spontaneous emission and stimulated emission of photons in a laser system? Discuss the main criteria that must be met to achieve laseraction. Which method is used to achieve this criteria? (5)

Some useful constants

1. Planck constant, $h = 6.626 \times 10^{-34}$ J.s
2. $\hbar = 1{,}05\times 10^{-34}$ J.s
3. Boltzmann constant, $K = 1.38 \times 10^{-23}$ $J.K^{-1}$
4. Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg
5. Charge of electron, $e = 1.6 \times 10^{-19}$ C
6. Speed of light in vacuum, $c = 3 \times 10^{8}$ $m.s^{-1}$
7. Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ $W.m^{-2}.K^{-4}$
8. Rest mass energy of electron = 512 KeV
9. Velocity of electron in free space= $3x10^{8}$ m/s

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(21) **Your Roll No.**.....

Sr. No. of Question Paper : 4812

Unique Paper Code : 32221403

Name of the Paper : Analog Systems and Applications

Name of the Course : **B.Sc. (Hons.) Physics-CBCS**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.
4. Use of scientific calculators is allowed.

1. Attempt any **five** of the following : (5×3=15)

(a) Define drift and diffusion currents in doped semiconductors.

(b) Explain the difference in physical mechanisms of avalanche and Zener breakdown in p-n. junction.

P.T.O.

(c) Draw I-V characteristics of the ideal diode and compare it with that of a practical diode, under forward bias and reverse bias conditions?

(d) A common emitter circuit has beta of 98, a collector current of 50 mA and base current of 500 μA. Calculate the reverse saturation current.

(e) Distinguish between Class A, Class B and Class C amplifiers with the help of load line and Q point.

(f) Define PIV, ripple factor and rectification efficiency in a rectifier circuit.

(g) An Op-Amp has a CMRR value of 55 dB and a differential mode gain of 1200. Find the common mode gain.

2. (a) For an abrupt p-n junction find the expression for potential V_B as a function of x for the case where N_a and N_d are of comparable magnitudes. Hence show that the barrier potential is given as follows :

$$V_B = q.N_a.N_d(W_p+W_n)^2 \ /2*\varepsilon(N_a + N_d)$$

wherein W_p and W_n are depletion widths on p and n sides respectively and all other symbols have their usual meaning. (10)

(b) Find the conductivity of a bar of pure Silicon of length 1 cm and cross-sectional area 1 mm^2 at 300K. Given $\mu_n = 0.13\ m^2/Vs$, $\mu_p = 0.05\ m^2/Vs$, $n_i = 1.5 \times 10^{16}\ m^{-3}$ and $e = 1.6 \times 10^{-19}$ C. (5)

3. (a) Explain the working of a center-tap full wave rectifier using suitable diagrams and obtain the expressions for (i) ripple factor and (ii) rectification efficiency. (10)

(b) What is a tunnel diode? Draw the I-V characteristics of the tunnel diode and briefly explain them. (5)

4. (a) Derive the stability factors for "voltage divider bias circuit" and "fixed bias circuit" and hence explain why "voltage divider bias circuit" is preferred over "fixed bias circuit". (10)

(b) Describe "DC load line" and "Q-point" of a transistor in CE configuration with appropriate diagram. (5)

5. (a) Draw the circuit diagram of a two stage RC coupled amplifier using transistors and also its frequency response curve. Why does the gain fall in low frequency range and high frequency range? (10)

P.T.O.

(b) What do you understand by the term small signal analysis? Draw the equivalent circuit in hybrid parameters for an n-p-n transistor in (i) CE configuration and (ii) CB configuration configurations. (5)

6. (a) Draw the circuit and explain the working of a 4-bit R-2R ladder network DAC using op- amp. In a 4-bit DAC, 0001 input results into 0.8 V output. What is the maximum output voltage of this DAC? (10)

(b) Draw the circuit of a voltage comparator using op-amp to give $+V_{sat}$ at the output if the input voltage is less than - 2V and $-V_{sat}$ for input more than +2V. (5)

7. (a) Draw the circuit of an Op-amp as a basic differentiator and find an expression for its output. Draw the output waveform when the input to the differentiator is a square wave. (10)

(b) A five -bit D/A converter produces an output of 9mV for a digital input of 10010. Find the output voltage for a digital input of 11011. Also find its full scale output voltage. (5)